Common Forces

Key Ideas

- Forces can be either long-range or contact interactions.
- A spring exerts a force that is proportional to its stretch or compression from its relaxed length, and is directed in such a way as to restore the spring to its relaxed length.
- Normal forces are contact forces between surfaces and are exerted perpendicular to the surface.
- Tension forces exerted by flexible cords, strings, or ropes are exerted along the length of the cord and are directed in the same way as a stretched spring.
- Friction forces are parallel to surfaces in contact and tend to resist the relative motion between surfaces.

Learning Objectives

After completing this section, you should be able to...

- distinguish between long-range and contact forces,
- calculate the magnitude and direction of a spring force, given its displacement from its relaxed length,
- determine the directions of normal and tension forces, and
- apply Newton's second law to solve problems involving a variety of forces.

Long-range and Contact Forces

Forces can be placed into two main categories: long-range and contact forces.

A **long-range force** is one that an object exerts on another over a distance, without the need for physical contact. An example is the gravitational force. Much later in your studies when you get to Section not found, you learn about the electrostatic force between electrically charged particles and the magnetic force due to electric currents and magnetic materials, like iron.

A **contact force** is one that is exerted when one object touches another. These types of forces are the ones most familiar to us in everyday life. In this section, we will describe a number of common contact forces that we will encounter when applying Newton's laws of motion to various situations.

Spring Force

If you pull either end of a coiled metal spring, you will find that the more it stretches from its original length, the more difficult it becomes to stretch. Similarly if you try to push the ends of the spring together, it becomes more difficult to push with greater compression. The **spring force** (or **elastic force**) exerted by a spring on whatever is in contact with it has two special properties. First, the magnitude of the force is directly proportional to the amount of stretch or compression of the spring. By "stretch" or "compression," we mean the difference in length between the relaxed length and the length when the force is applied. Second, the force always acts in the direction opposite to that of the stretch or compression. Mathematically, the behavior of the force due to a spring is given by Hooke's law.

Hooke's Law

Let $\vec{x}$ be a position vector that points from one end of a spring to the other end where an object is attached. Let $\vec{x}_r$ be the same position vector when the spring is at its relaxed length. Then, the force that a spring applies to that object is given by

$$\vec{F}_s = -k\Delta\vec{x} \tag{5.7}$$

where $\Delta\vec{x} = \vec{x} - \vec{x}_r$ is the displacement vector, and k is the **spring constant**, a property of the spring that measures the spring's stiffness. The SI units for the spring constant are N/m.

A larger value of the spring constant k means a stiffer spring; that is, the larger the spring constant, the larger the force given the same magnitude of displacement. A spring with a larger k is more difficult to stretch or compress.

Figure 5.15 illustrates how Hooke's law works. A spring is attached to a wall at its left end, where it is fixed in place, and to a block that is free to move at its right end. In this scenario, we are assuming that the idealized surface does not exert any force on the block in the x-direction. We are interested in finding the force that the spring applies to the block. Initially, the spring is relaxed, as shown in Figure 5.15(a). Vector $\vec{x}_r$ points from the left end of the spring to the right. When the spring is in the relaxed position, vector $\vec{x}$ is identical to vector $\vec{x}_r$. Because $\vec{x} = \vec{x}_r$, the displacement $\Delta\vec{x} = \vec{x} - \vec{x}_r = 0$, so the force by the spring on the block $\vec{F}_{s\text{ on }b} = 0$. That is, when the spring is at its relaxed length, it does not apply a force to the block.

Next, the block is moved to the left of the relaxed position, so that the spring is compressed, as shown in Figure 5.15(b). Note that the position vector $\vec{x}$ is now shorter than $\vec{x}_r$, so the displacement $\Delta\vec{x} = \vec{x} - \vec{x}_r$ points to the left. The negative sign in Hooke's law tells us that the force vector always points in the opposite direction of the displacement. So, the force of the spring on the block $\vec{F}_{s\text{ on }b}$ points to the right.

Similarly, when the block is moved to the right of the relaxed position, stretching the spring, the position vector $\vec{x}$ is longer than $\vec{x}_r$, so the displacement $\Delta\vec{x} = \vec{x} - \vec{x}_r$ points to the right. The force by the spring on the block $\vec{F}_{s\text{ on }b}$ therefore points to the left, as shown in Figure 5.15(c). Note how the force applied by the spring always acts in a direction that tends to restore the spring to its original relaxed length. For this reason, the spring force is an example of what is called a **restoring force**.

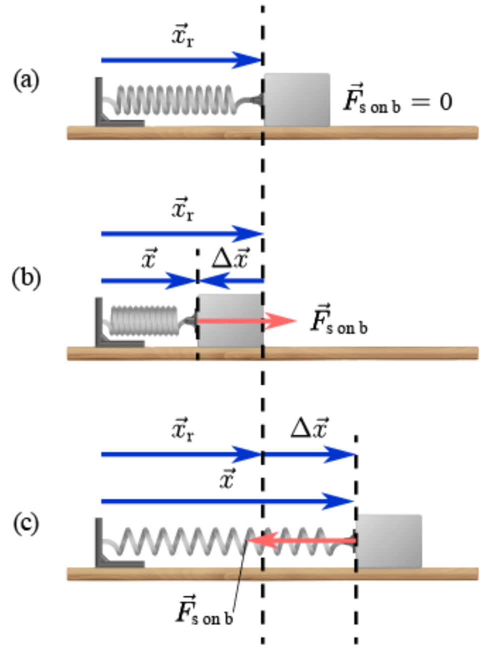

Figure 5.15 A spring exerts its force proportional to a displacement, whether it is compressed or stretched. (a) The spring is in a relaxed position and exerts no force on the block. (b) The spring is compressed by a displacement $\Delta\vec{x}$ to the left, and the spring exerts a force on the block $\vec{F}_{s\ on\ b}$ to the right. (c) The spring is stretched by a displacement $\Delta\vec{x}$ to the right, and the spring exerts a force on the block $\vec{F}_{s\ on\ b}$ to the left.

But what about the left end of the spring that is attached to a fixed wall? At first glance, it might seem that the left end applies no force to the wall because it does not move, but in fact it does apply a force, which we can determine by carefully applying the same definitions of $\vec{x}$, $\vec{x}_r$, and $\Delta\vec{x}$, as we did above. This is done in Figure 5.16, where the only differences from Figure 5.15 are that the position and displacement vectors are reversed. Since we are now interested in the force the spring applies to the wall, we now define $\vec{x}$ and $\vec{x}_r$ to point from the right end of the spring toward the left. We now get the force of the spring on the wall, $\vec{F}_{s\ on\ w}$, that is again always in the opposite direction of the displacement $\Delta\vec{x}$.

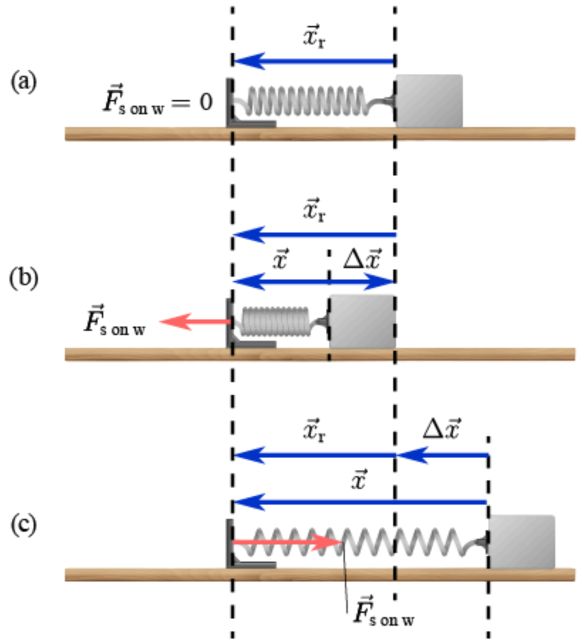

Figure 5.16 To find the force the spring applies to the wall, we draw the position and displacement vectors in directions opposite to the ones in Figure 5.15. (a) The spring is in a relaxed position and exerts no force on the wall. (b) The displacement vector, with respect to the right end of the spring, points right. So, the force the spring exerts on the wall, $\vec{F}_{s\ on\ w}$, points toward the left. (c) The displacement vector, with respect to the right end of the spring, points left. So, the force by the spring on the wall $\vec{F}_{s\ on\ w}$ is now toward the right.

We can now see that spring does exert a force on the wall, and in general, as long as a spring is stretched or compressed, it exerts forces on both ends. Unlike the block, the wall is not affected by this force because it is extremely massive, and so from Newton's second law, its acceleration is effectively zero. While the direction of the displacement $\Delta\vec{x}$ changes depending on which end of the spring we analyze, its magnitude does not. Therefore, the force that a spring exerts on an object at one end is the same magnitude as the force it exerts on an object at its other end, but opposite in direction.

The result that a compressed or stretched spring exerts equal force magnitude at either of its ends is actually an idealization that applies only to cases where we can safely ignore the mass of the spring. For most situations we analyze, we will safely make the assumption of "massless" springs. This result will also apply to our discussion of tension forces in flexible cords below.

Often when analyzing springs, since they typically align with one dimension, we can just use the scalar component of the spring force. Hooke's law can then be written as:

$$F_{s,x} = -k\Delta x \tag{5.8}$$

In many cases, the scalar displacement Δx can be easily determined. However, to avoid confusion, you can always resort to drawing $\vec{x}$, $\vec{x}_r$, and $\Delta\vec{x}$ as described above to find the correct magnitude and direction of the spring force on an object.

Example 5.10

Calculating a Spring Force

The relaxed length of the spring in Figure 5.15 (b) is $12.0\ \mathrm{cm}$, and its current length is $7.2\ \mathrm{cm}$. If the spring constant is $50\ \mathrm{N/m}$, what is the magnitude of the force of the spring on the block?

Strategize

We can use Hooke's law in the form of Equation 5.8. The relaxed and compressed lengths are given, so we can calculate the displacement. From the displacement and spring constant we can find the force. Assume the direction of the position vector x_r gives the positive x-direction, which is toward the right in Figure 5.15.

Develop and Solve

Converting units to meters, we therefore have $x_r = 0.120\ \mathrm{m}$ and $x = 0.072\ \mathrm{m}$.

Using Equation 5.8, the spring force is then:

$$\begin{aligned} F_{s,x} &= -k\Delta x \\ &= -(50\ \mathrm{N/m})(0.072\ \mathrm{m} - 0.120\ \mathrm{m}) \\ &= 2.4\ \mathrm{N} \end{aligned}$$

This result is the x-component of the force, but since the force only has a nonzero x-component, the force magnitude on the block is $F_s = 2.4\ \mathrm{N}$ as well.

Assess

Note the displacement is negative, but this negative sign cancels with the negative sign in Hooke's law, resulting in a force component in the positive x-direction, which is what we expected from Figure 5.15 (b). Be careful with signs when using Hooke's law. In this case, we were only interested in finding the magnitude of the force, so we could have taken the absolute value of Equation 5.8, where $F_s = |-k\Delta x| = k\,|\Delta x|$, and $|\Delta x|$ is the absolute value of the displacement. When in doubt, you can always find the magnitude of the spring force, then determine its direction physically, based on whether it is stretched or compressed. This is another example of why it is useful to always draw a picture of the situation.

Hooke's law is an important model not only for the behavior of actual springs, but also for many other spring-like forces. The forces between atoms in atomic bonds, for example, behave like springs, with a larger restoring force being applied as atoms move farther away from their equilibrium position. Spring-like forces are critical for describing oscillations and waves, two important phenomena we will examine in Chapter 15 (Section not found) and Chapter 16 (Section not found).

Normal Forces

When surfaces of objects come into contact, each surface exerts a force on the other. The direction of the force that a surface applies is typically broken into two components. One component, parallel to the surface, is called the friction force.